Circuits Problem Set

9.1)

When examining signal transmission in parallel conductors, we must consider the electromagnetic coupling that occurs between them. In twisted pair cables, the twisting pattern serves a critical function beyond simple mechanical stability.

The fundamental principle at work is that of mutual inductance between parallel conductors. When current flows in opposite directions through parallel wires, a magnetic field is generated between them. This field can induce unwanted signals,a phenomenon known as crosstalk.

By twisting the pairs, each small segment of wire experiences an equal but opposite electromagnetic influence from adjacent segments, effectively canceling out external interference. This creates an effect similar to a Faraday cage, where electromagnetic fields are contained and controlled.

The shielding that often surrounds these twisted pairs serves two complementary purposes:

- It creates a barrier against external electromagnetic interference and radio frequency interference.
- It prevents internal electromagnetic signals from radiating outward and potentially interfering with nearby sensitive electronic systems

This dual protection is particularly important in environments with multiple signal-carrying cables or where electromagnetic compatibility is critical.

9.2)

The skin depth can be derived from Maxwell's equations by examining how electromagnetic waves propagate in conductive media. The resulting expression is:

$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \frac{1}{\sqrt{\pi\nu\mu\sigma}}$

Where:

- ν represents frequency (Hz)
- μ represents magnetic permeability (H/m)
- σ represents electrical conductivity (S/m)

For our specific case with salt water having conductivity of approximately 4 S/m at a frequency of 10^4 Hz:

$\delta = \frac{1}{\sqrt{\pi \cdot 10^4 \text{ Hz} \cdot 4\pi \cdot 10^{-7} \text{ H/m} \cdot 4 \text{ S/m}}}$

The magnetic permeability term $4\pi \cdot 10^{-7}$ represents μ_0, the permeability of free space.

Evaluating this expression: $\delta = \frac{1}{\sqrt{\pi \cdot 10^4 \cdot 4\pi \cdot 10^{-7} \cdot 4}} = 2.52$ mm

9.3)

The Poynting vector $\vec{P}$ represents the directional energy flux density of an electromagnetic field. It is defined as the cross product of the electric field $\vec{E}$ and the magnetic field $\vec{H}$:

$\vec{P} = \vec{E} \times \vec{H}$

For a coaxial cable, we can analyze this in cylindrical coordinates. The electric field points radially outward from the center conductor, while the magnetic field forms concentric circles around it.

In a coaxial geometry with inner radius r_1 and outer radius r_2, the electric field at a distance r from the center is: $\vec{E} = \frac{V}{r\ln(r_2/r_1)}\hat{r}$

The magnetic field at the same point is: $\vec{H} = \frac{I}{2\pi r}\hat{\phi}$

Taking their cross product: $\vec{P} = \frac{V}{r\ln(r_2/r_1)}\hat{r} \times \frac{I}{2\pi r}\hat{\phi} = \frac{VI}{2\pi r^2 \ln(r_2/r_1)}\hat{z}$

To find the total power flowing through the cable, we integrate over the cross-sectional area: $P = \iint_{\text{cross-section}} \vec{P} \cdot d\vec{A} = \int_0^{2\pi} \int_{r_1}^{r_2} \frac{VI}{2\pi r^2 \ln(r_2/r_1)} r dr d\theta$

Simplifying: $P = \frac{VI}{\ln(r_2/r_1)} \int_{r_1}^{r_2} \frac{1}{r} dr = \frac{VI}{\ln(r_2/r_1)} \ln\left(\frac{r_2}{r_1}\right) = VI$

9.4)

The characteristic impedance of a transmission line represents the ratio of voltage to current for a wave propagating along the line. For a transmission line consisting of two parallel conductors with width w and separation h, we need to analyze the electromagnetic fields.

Starting with the relationship between impedance, inductance, and capacitance: $Z = \sqrt{\frac{L}{C}}$

Where L is the inductance per unit length and C is the capacitance per unit length.

For parallel conductors, we can determine the magnetic field using Ampere's law. A current I flowing through a conductor creates a magnetic field: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

For a uniform magnetic field between the conductors: $B = \frac{\mu_0 I}{h}$

The magnetic flux linking the conductors is: $\Phi = \int \vec{B} \cdot d\vec{A} = \mu_0 \frac{I}{h} \cdot h = \mu_0 I$

Therefore, the inductance per unit length is: $L = \frac{\Phi}{I} = \frac{\mu_0 h}{w}$

For the capacitance, we use the relationship between electric field and potential: $V = \int \vec{E} \cdot d\vec{s}$

For a uniform electric field between parallel plates: $E = \frac{V}{h}$

The capacitance per unit length is: $C = \frac{Q}{V} = \frac{\varepsilon_0 \varepsilon_r w}{h}$

Substituting these expressions into the impedance formula: $Z = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu_0 h/w}{\varepsilon_0 \varepsilon_r w/h}} = \sqrt{\frac{\mu_0 h^2}{\varepsilon_0 \varepsilon_r w^2}} = \frac{h}{w}\sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}}$

The wave velocity can be determined from: $v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}} = \frac{c}{\sqrt{\varepsilon_r}}$

Where c is the speed of light in vacuum. This shows that the wave propagates at a fraction of the speed of light, determined by the relative permittivity of the medium.

9.5)

9.5a)

For a coaxial cable with inner radius r_1, outer radius r_2, and dielectric with relative permittivity ε_r, the characteristic impedance is:

$Z = \frac{1}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r}}\ln\left(\frac{r_2}{r_1}\right)$

Given:

- $\varepsilon_r = 2.26$
- $r_1 = 0.406$ mm
- $r_2 = 1.48$ mm

Substituting these values:

$Z = \frac{1}{2\pi}\sqrt{\frac{4\pi\times10^{-7}}{8.854\times10^{-12}\times2.26}}\ln\left(\frac{1.48}{0.406}\right)$

$Z = \frac{1}{2\pi}\sqrt{\frac{4\pi\times10^{-7}}{2.001\times10^{-11}}}\ln(3.645)$

$Z = \frac{1}{2\pi}\times 120\pi \times 0.377 = 51.6\Omega$

9.5b)

The velocity of wave propagation in the coaxial cable is:

$v = \frac{1}{\sqrt{\mu_0\varepsilon_0\varepsilon_r}} = \frac{c}{\sqrt{\varepsilon_r}}$

Where c is the speed of light in vacuum (3×10^8 m/s).

$v = \frac{3\times10^8}{\sqrt{2.26}} = 1.99\times10^8$ m/s

This represents approximately 66% of the speed of light in vacuum, demonstrating how the dielectric material slows electromagnetic wave propagation.

9.5c)

For a signal traveling at velocity v during time t, the distance covered is:

$d = v \times t = 1.99\times10^8 \text{ m/s} \times 1\times10^{-9} \text{ s} = 0.199 \text{ m}$

9.5d)

To maintain the same characteristic impedance with a different outer radius, the ratio r_2/r_1 must remain constant.

Given that $r_2 = 20$ mil in the new configuration:

$\frac{1.48}{0.406} = \frac{20 \text{ mil}}{r_1}$

$r_1 = \frac{20 \text{ mil}}{3\ 645} = 5.23$ mil

9.5e)

The relationship between wavelength λ, frequency f, and wave velocity v is:

$\lambda = \frac{v}{f}$

Rearranging to find frequency:

$f = \frac{v}{\lambda} = \frac{1.99\times10^8 \text{ m/s}}{1.48\times10^{-2} \text{ m}} = 1.346\times10^{10} \text{ Hz} = 13.46 \text{ GHz}$

9.6b)

When a transmission line with characteristic impedance Z_0 is connected to a load with impedance Z_L, the reflection coefficient Γ quantifies how much of the incident signal is reflected back:

$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

In our scenario, the source impedance is mismatched with the effective load impedance by a factor of 2: $Z_0 = 51.6\Omega$ and $Z_L = 25.8\Omega$

Calculating the reflection coefficient:

$\Gamma = \frac{25.8-51.6}{25.8+51.6} = \frac{-25.8}{77.4} = -\frac{1}{3}$

The negative sign indicates a phase reversal in the reflected wave. The magnitude $|\Gamma| = \frac{1}{3}$ means that one-third of the incident signal power is reflected back due to the impedance mismatch.